Montestigliano Workshop | **Problem 9: Geometric modeling with RBFs: Part II** | **Level 2, Grady Wright**

This is a follow up to problem 3 where you developed a method for doing geometric modeling of surfaces. In this problem you will compute some properties of surfaces from your RBF geometric model. These include the surface tangent vectors, surface normal vectors, and mean curvature, which are important in applications (see, for example, [2]). As these quantities all involve derivatives of the underlying parametric equation for the surface, this problem is also related to problem 4.

As in Problem 3, let $\mathbf{x}(\lambda,\theta)$ denote the parametric representation for the object computed from an RBF interpolant of each component. Letting

$$\boldsymbol{\tau}^{\lambda} := \frac{\partial}{\partial\lambda}\mathbf{x}(\lambda,\theta) = \left(\frac{\partial}{\partial\lambda}x(\lambda,\theta), \frac{\partial}{\partial\lambda}y(\lambda,\theta), \frac{\partial}{\partial\lambda}z(\lambda,\theta)\right), \tag{1}$$

$$\boldsymbol{\tau}^{\theta} := \frac{\partial}{\partial\theta}\mathbf{x}(\lambda,\theta) = \left(\frac{\partial}{\partial\theta}x(\lambda,\theta), \frac{\partial}{\partial\theta}y(\lambda,\theta), \frac{\partial}{\partial\theta}z(\lambda,\theta)\right), \tag{2}$$

the unit tangent vectors to $\mathbf{x}(\lambda,\theta)$ are then given by

$$\hat{\boldsymbol{\tau}}^{\lambda} := \frac{\boldsymbol{\tau}^{\lambda}}{\|\boldsymbol{\tau}^{\lambda}\|} \quad \text{and} \quad \hat{\boldsymbol{\tau}}^{\theta} := \frac{\boldsymbol{\tau}^{\theta}}{\|\boldsymbol{\tau}^{\theta}\|}, \tag{3}$$

while the unit normal vector is given by

$$\hat{\boldsymbol{\eta}} := \frac{\boldsymbol{\tau}^{\lambda}\times\boldsymbol{\tau}^{\theta}}{\|\boldsymbol{\tau}^{\lambda}\times\boldsymbol{\tau}^{\theta}\|}. \tag{4}$$

The mean H curvature of the surface, and can be computed as [1, §16.5]

$$H = \frac{eG - 2fF + gE}{2(EG - F^2)}, \tag{5}$$

where E, F, and G are coefficients of the first fundamental form,

$$E = \boldsymbol{\tau}^{\lambda}\cdot\boldsymbol{\tau}^{\lambda},\ F = \boldsymbol{\tau}^{\lambda}\cdot\boldsymbol{\tau}^{\theta},\ G = \boldsymbol{\tau}^{\theta}\cdot\boldsymbol{\tau}^{\theta}, \tag{6}$$

and e, f, and g are coefficients of the second fundamental form,

$$e = \left(\frac{\partial}{\partial\lambda}\boldsymbol{\tau}^{\lambda}\right)\cdot\hat{\boldsymbol{\eta}},\ f = \left(\frac{\partial}{\partial\theta}\boldsymbol{\tau}^{\lambda}\right)\cdot\hat{\boldsymbol{\eta}},\ g = \left(\frac{\partial}{\partial\theta}\boldsymbol{\tau}^{\theta}\right)\cdot\hat{\boldsymbol{\eta}}. \tag{7}$$

$$\tag{8}$$

Mean curvature shows up in some elasticity models for membranes.

(i) Write Matlab functions for computing the tangent and normal vectors from an RBF geometric model of a surfaces. Also write a function for computing the mean curvature. Test your method on a known surface (say the sphere, or a more general ellipsoid).

(ii) Repeat 3.a) for the Bumpy Sphere, however, in the plot of the surface, color the bumpy sphere according to its mean curvature.

References

[1] A. Gray, *Modern Differential Geometry of Curves and Surfaces with Mathematica*, CRC Press, Boca Raton, FL, 1997.

[2] V. Shankar, G. B. Wright, A. L. Fogelson, and R. M. Kirby, *A study of different modeling choices for simulating platlets within the immersed boundary method*, Appl. Num. Math., 63 (2013), pp. 58–77.